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STATISTICAL ANALYSIS OF THE STABILITY OF IMPERFECT CYLINDRICAL --ETC(U)
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STATISTICAL ANALYSIS OF THE STABILITY OF IMPERFECT CYLINDRICAL SHELLS

by

B.P. Makarov

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А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

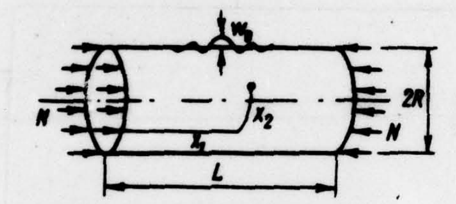
Russian	English
rot	curl
lg	log

STATISTICAL ANALYSIS OF THE STABILITY OF IMPERFECT CYLINDRICAL SHELLS

B. P. Makarov (Moscow)

This work examines the problem of stability of a thin cylindrical shell, the middle surface of which has random initial deviations from the ideal shape. For a solution, we used the method of V. V. Bolotin [1]. For the beginning, we use statistical characteristics of initial errors obtained according to experimental data in work [2]. We are trying to construct a solution to the stochastic problem of stability of a cylindrical shell with a random calculated number of random parameters which characterize the deviations of the middle surface from the ideal shape. We selected for these parameters coefficients of a double Fourier series, with the aid of which the errors of initial errors are presented.

Let us first examine the deterministic problem of stability of a cylindrical shell which has initial deviation from the ideal shape. Let us assume that the shell is under the influence of longitudinal compressing load N (figure). We will consider that the initial and additional sagging (w_0 and w_1) are small in comparison with the thickness of the shell (h), and the stress-deformation state quickly changes.



Additional movement of w_1 in the precritical stage with these suppositions must satisfy equations of the Donnell-Mushtar-Vlasov types

$$D\Delta\Delta w_1 = \frac{1}{R}\frac{\partial^2 \chi}{\partial x_1^2} - N\frac{\partial^2 (w_1 + w_0)}{\partial x_1^2}, \quad \frac{1}{Eh}\Delta\Delta\chi_1 = -\frac{1}{R}\frac{\partial^2 w_1}{\partial x_1^2} \qquad (1)$$

Here χ_1 - function of additional stresses, E - modulus of elasticity, D - cylindrical rigidity, R - radius of the shell. Through Δ the Laplas operator is designated.

Switching to the solution equation, we obtain

$$D\Delta\Delta\Delta\Delta w_1 + \frac{Eh}{R^2}\frac{\partial^4 w_1}{\partial x_1^4} + N\Delta\Delta\left(\frac{\partial^2 w_1}{\partial x_1^2}\right) = -N\Delta\Delta\left(\frac{\partial^2 w_0}{\partial x_1^2}\right) \qquad (2)$$

As boundary conditions, we take support conditions:

$$w_1 = \frac{\partial^2 w_1}{\partial x_1^2} = 0 \qquad (x_1 = 0, L) \qquad (3)$$

Here, L - length of the shell. Tangential conditions will be formed thusly:

$$N_{11} = -N, N_{22} = 0 \; (x_1 = 0, L) \qquad (4)$$

Let us assume that deviations of the middle surface of the shell from the ideal cylindrical shape can be presented in the form of a Fourier series:

$$w_0(x_1, x_2) = \sum_m g_m^{(0)} \sin \frac{m\pi x_1}{L} + \sum_m \sum_n g_{mn}^{(1)} \sin \frac{m\pi x_1}{L} \sin \frac{nx_2}{R} + \\ + \sum_m \sum_n g_{mn}^{(2)} \sin \frac{m\pi x_1}{L} \cos \frac{nx_2}{R} \tag{5}$$

Here, $g_m^{(0)}, g_{mn}^{(1)}, g_{mn}^{(2)}$ - coefficients of breakdown which will subsequently be examined as random.

We will search for a solution in the form of a series similar to (5) and satisfies the boundary conditions of the problem

$$w_1(x_1, x_1) = \sum_m f_m^{(0)} \sin \frac{m\pi x_1}{L} + \sum_m \sum_n f_{mn}^{(1)} \sin \frac{m\pi x_1}{L} \sin \frac{nx_2}{R} + \\ + \sum_m \sum_n f_{mn}^{(2)} \sin \frac{m\pi x_1}{L} \cos \frac{nx_2}{R} \tag{6}$$

For coefficients we obtain expression

$$f_m^{(0)} = \frac{g_m^{(0)}\gamma_m}{1-\gamma_m}, \quad f_{mn}^{(1)} = \frac{g_{mn}^{(1)}\gamma_{mn}}{1-\gamma_{mn}}, \quad f_{mn}^{(2)} = \frac{g_{mn}^{(2)}\gamma_{mn}}{1-\gamma_{mn}} \tag{7}$$

Here,

$$\gamma_m = \frac{N}{N_m}, \qquad \gamma_{mn} = \frac{N}{N_{mn}}, \qquad \lambda_m = \frac{m\pi R}{L}$$

$$N_m = \frac{Eh^2}{R}\left[\frac{1}{12(1-\mu^2)}\frac{h}{R}\lambda_m^2 + \frac{R}{h}\frac{1}{\lambda_m^2}\right]$$

$$N_{mn} = \frac{Eh^2}{R}\left[\frac{1}{12(1-\mu^2)}\frac{h}{R}\frac{(n^2+\lambda_m^2)^2}{\lambda_m^2} + \frac{R}{h}\frac{\lambda_m^2}{(n^2+\lambda_m^2)^2}\right] \tag{8}$$

Using the solution in (6), we can determine the precritical stress in the shell. Thus, for example, the surrounding stresses in the middle surface will be equal to

$$\frac{N_{22}}{h} = -\frac{E}{R}\left[\sum_m f_m^{(0)} \sin \frac{m\pi x_1}{L} + \sum_m \sum_n f_{mn}^{(1)} \frac{\lambda_m^4}{(n^2+\lambda_m^2)^2} \sin \frac{m\pi x_1}{L} \sin \frac{nx_2}{R} + \\ + \sum_m \sum_n f_{mn}^{(2)} \frac{\lambda_m^4}{(n^2+\lambda_m^2)^2} \sin \frac{m\pi x_1}{L} \cos \frac{nx_2}{R}\right] \tag{9}$$

Let us set up a problem on the stability of solution (6). The equation of neutral equilibrium will be obtained by linearization near the undisturbed equilibrium. Here, as this is done normally in the theory of elastic stability, we will consider the system "rigid",

disregarding the precritical movements and initial deviations in comparison with the deviations from undisturbed equilibrium w_*. As experimental data show, for shells which do not have specially created defects, this linearization is fully right. The equation for deviations w_* take the form:

$$D\Delta\Delta\Delta\Delta w_* + \frac{Eh}{R^2}\frac{\partial^4 w_*}{\partial x_1^4} = \Delta\Delta\left(N_{\alpha\beta}\frac{\partial^2 w_*}{\partial x_\alpha \partial x_\beta}\right) \tag{10}$$

Precritical surrounding stress in the middle surface, determined according to formula (9), enter into the right half of equation (10). For stresses $N_{\alpha\beta}$ we have expression:

$$N_{11} = -N + \frac{\partial^2 \chi_1}{\partial x_2^2}, \quad N_{22} = \frac{\partial^2 \chi_1}{\partial x_1^2}, \quad N_{12} = N_{21} = -\frac{\partial^2 \chi_1}{\partial x_1 \partial x_2} \tag{11}$$

Let us note that a similar approach was earlier used by S. N. Kahn [3] for an analysis of the stability of shells which have axisymmetrical errors.

An exact analysis of the equation of neutral equilibrium (10) is difficult due to the presence in the right half of multiplier $N_{\alpha\beta}$. Therefore, for determining the critical forces, we will use a variation of the Bubnov-Galerkin method.

Let us find function w_* in the form of an expression which satisfies conditions on the edges of the shell:

$$w_* = f_2 \sin\frac{j\pi x_1}{L}\sin\frac{kx_2}{R} + f_3 \sin\frac{j\pi x_1}{L}\cos\frac{kx_2}{R} \tag{12}$$

where j and k - wave numbers which characterize the form of loss of stability. Expression (12) was selected with consideration of the shift of phase in the surrounding direction.

Let us place (12) into equation (10) and demand it so that it will be satisfied in terms of the Bubnov-Galerkin method. Setting to zero in order to determine the systems of equations with respect to f_1 and f_2, we obtain a relationship which connects the critical value of the load parameter with the coefficients of breakdown of the function of initial errors

$$\left[1-\frac{v}{v_{jk}}-\sum \xi_m^{(0)}\frac{a_m}{v_m(1-\gamma_m)}\right]^2 = \\ =\left[\sum_m \xi_{mn_*}^{(1)}\frac{b_m}{v_{mn_*}(1-\gamma_{mn_*})}\right]^2+\left[\sum_m \xi_{mn_*}^{(2)}\frac{b_m}{v_{mn_*}(1-\gamma_{mn_*})}\right]^2 \tag{13}$$

Here

$$\xi_m^{(0)}=\frac{g_m^{(0)}}{h},\quad \xi_{mn_*}^{(1)}=\frac{g_{mn_*}^{(1)}}{h},\quad \xi_{mn_*}^{(2)}=\frac{g_{mn_*}^{(2)}}{h} \\ v=\frac{Eh^2}{R}N,\quad v_{jk}=\frac{1}{12(1-\mu^2)}\frac{h}{R}\frac{(k^2+\lambda_j^2)^2}{\lambda_j^2}+\frac{R}{h}\frac{\lambda_j^2}{(k^2+\lambda_j^2)^2} \\ n_*=2k,\ v_{mn_*}=v_{mn}|_{n_*=2k},\ \gamma_{mn_*}=\gamma_{mn}|_{n_*=2k} \tag{14}$$

Coefficients a_m and b_m depend on the wave numbers.

Relationship (13) can be significantly simplified if we consider the following inequality:

$$\gamma_m \ll 1,\ \gamma_{mn_*} \ll 1 \tag{15}$$

In this case, for v, we obtain an explicity expression

$$v=v_{jk}/(1+X_0+\sqrt{X_1^2+X_2^2}) \tag{16}$$

where

$$X_0=\sum_m \frac{\xi_m^{(0)}a_m}{v_m},\quad X_1=\sum_m \frac{\xi_{mn_*}^{(1)}b_m}{v_{mn_*}},\quad X_2=\sum_m \frac{\xi_{mn_*}^{(2)}b_m}{v_{mn_*}} \tag{17}$$

Let us note that in the expression for v, the wave numbers j and k remained random. For their determination in the future, we must minimize v.

Let us employ the approximation relationship (16) for studying the statistical tie between critical stresses and random deviations from the ideal shape.

We will treat breakdown in (5) as a spectral presentation of random function w_0. Then the set of values $\xi_m^{(0)}, \xi_{mn_*}^{(1)}, \xi_{mn}^{(2)}$ will describe the random spectrum for w_0. Let us assume that these coefficients form the system of normal random values. In this case, the probability functions for coefficients X_0, X_1, and X_2 are compositions of normal laws. For their determination, it is sufficient to have a correlation matrix of the spectrum of initial errors.

We must determine the mathematical expectation of critical load. Averaging relationship (16), we obtain in the first approximation

$$\langle v \rangle = v_{jk}/(1 + \langle X_0 \rangle + \langle \sqrt{X_1^2 + X_2^2} \rangle) \tag{18}$$

The mathematical expectations in the right part are expressed through the characteristics of the coefficients from series (5) in the following manner:

$$\langle X_0 \rangle = \sum_m \langle \xi_m^{(0)} \rangle \frac{a_m}{v_m}$$

$$\langle X_1 \rangle = \sum_m \langle \xi_{mn_*}^{(1)} \rangle \frac{b_m}{v_{mn_*}}, \qquad \langle X_2 \rangle = \sum_m \langle \xi_{mn_*}^{(2)} \rangle \frac{b_m}{v_{mn_*}} \tag{19}$$

$$\langle \sqrt{X_1^2 + X_2^2} \rangle = c \iint \sqrt{X_1^2 + X_2^2} \exp\left\{-\frac{1}{2}\frac{1}{1 - w_{12}^2} \times \right.$$
$$\left. \times \left[\frac{(X_1 - \langle X_1 \rangle)^2}{\sigma_1^2} - \frac{2w_{12}(X_1 - \langle X_1 \rangle)(X_2 - \langle X_2 \rangle)}{\sigma_1\sigma_2} + \frac{(X_2 - \langle X_2 \rangle)^2}{\sigma_2^2}\right]\right\} dX_1\, dX_2$$

Dispersions σ_1^2 and σ_1^2, and also coefficient of correlation w_{12}, represent linear combinations from the elements of the correlation matrix of the spectrum of initial errors.

A particularly simple result is obtained in the case where function w_0 is uniform in the circular direction. The expression for [v] takes the form:

$$\langle v \rangle = v_{jk}/(1 + \langle X_0 \rangle + \sqrt{2}\langle |X_1| \rangle) \tag{20}$$

In the final stage, we must minimize value [v] according to the wave numbers j and k, accomplishing a computation for the wave range which corresponds to the approximate shape of loss in stability.

Relationships like (16) can be used also for determining the probability density of critical load. Following work [1], we obtain the following expression for p(v):

$$p(v)=c\iint p\,[g(v,\ X_1,\ X_2)\,X_1,\ X_2]\left|\frac{\partial g\,(vX_1,\ X_2)}{\partial v}\right| dX_1 dX_2 \qquad (21)$$

where c - normalizing multiplier, and function $g(v, X_1, X_2)$ equals

$$g\,(v,\ X_1,\ X_2)=v_{jk}/v-1-\sqrt{X_1^2-X_2^2}$$

Let us briefly stop at some numerical results. Let us examine, for example, the series of cylindrical shells, for which similar experimenatal investigations were conducted in work [2]. Nominal values of parameters were as follows:

$$h=0.018\ cm,\ R=10\ cm,\ L=30\ cm,\ E=2.1\cdot10^6\ kg/cm^2$$

The computations, conducted with various values of wave numbers, provided the following results:

$k=3$	4	5	6	7	8	
$\langle v\rangle=1.12$	0.92	0.65	0.42	0.44	0.31	$(\lambda_j/k=1)$
$\langle v\rangle=1.10$	1.21	0.95	0.81	0.70	0.58	$(\lambda_j/k=1,5)$

The smallest value [v] corresponds to k=8, $\lambda_j/k=1$. According to the data in the experiment, $[v_*]=0.23$.

The difference between the theoretical and experimental values was 30%, which attests to the satisfactory congruence of results.

For constructing the distribution of critical forces, we can use relationship (21). Integration in the form of (21) is easiest to accomplish numerically.

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